Investigation of the Ratio Between Phase and Group Birefringence in Optical Single-Mode Fibers

M. Legre, M. Wegmuller, and N. Gisin

***Abstract*—A measurement technique for the phase and group birefringence of an optical fiber is thoroughly investigated. It is based on differential group delay measurements of twisted fibers and is capable of giving in a simple and elegant way the intrinsic birefringence values in the absence of twist. Analyzing various fibers with this method, we find that phase and group birefringence can be quite different for certain fiber types. Consequently, the commonly used assumption that in an optical fiber, phase and group birefringence are equal—and the resulting carelessness in distinguishing between these two a priori separate physical effects—is to be employed cautiously.**

***Index Terms*—Beatlength, birefringence, differential group delay (DGD), optical fiber measurements, optical fiber properties.**

I. Introduction

THE polarization properties of a single-mode optical fiber are characterized by its local phase and group birefringences and by the amount and statistics of the energy transfer between the two polarization modes. Concentrating on short distances where statistical polarization mode coupling can be neglected, it is frequently assumed that there is a simple relation between the phase birefringence (characterized by the beatlength L_b) and the group birefringence [characterized by the (local) differential group delay (DGD)]

$$\mathrm{DGD} = \frac{\lambda}{c \cdot L_b} \qquad (1)$$

where DGD is in ps/m, λ is the (center) wavelength of the light source, and c ($= 300$ μm/ps), the velocity of light in vacuum. It is easy to show that (1) can also be expressed as

$$\frac{\delta\beta}{\omega} = \frac{d}{d\omega}\delta\beta = \delta\beta' \qquad (2)$$

where $\delta\beta$ is the phase birefringence (in rad/m). Equation (1) is therefore equivalent with the assumption that the phase delay ($\delta\beta/\omega$) and group birefringence (both in units of s/m) are equal.

This seems to hold fairly well for standard fibers typically employed in telecom links. However, going back to the early days of polarization maintaining (PM) fiber development, one finds examples [1], [2] of fibers with beatlengths in the millimeter range where the group birefringence can be as much as three times larger than the phase delay.

As a consequence, we were interested in the ratio of phase delay to group birefringence of more recently developed fibers, of fibers with intermediate phase birefringence values, and of special fibers such as photonic crystal or Er-doped fibers.

In order to measure the correct ratio (especially for fibers with low phase birefringence), one has to be careful that the fiber is not subjected to twist. These "twistless" values of interest are readily obtained by a special measurement method presented in Section II. Due to its elegance and simple applicability, the measurement method is also a viable alternative to more standard methods for beatlength and DGD determination. Therefore, in Section III, we investigate its precision by comparing the results for various fibers with those obtained from more standard methods, and discuss the range of fibers suited for analysis by our method. Then, in Section IV, we give the ratio of phase delay to group birefringence for a variety of different fibers, along with some intuitive explanation for our findings. Section V summarizes this paper.

II. Description of Measurement Method

It is well known that twisting a fiber induces circular birefringence on it. Consequently, its polarization properties change [3], leading to elliptical or even circular principal states for the case of very large twist. Both the phase birefringence $\delta\beta$ (characterized by the beatlength $L_b = 2\pi/\delta\beta$) and the group birefringence $\delta\beta'$ change with the twist rate. For a constant, homogeneous twist along the fiber, Siddiqui *et al.* have found in [4] that the DGD becomes[1]

$$\mathrm{DGD} = \frac{\delta\beta_L \cdot \delta\beta'_L + (\delta\beta_C - 2\gamma) \cdot \delta\beta'_C}{\sqrt{\delta\beta_L^2 + (\delta\beta_C - 2\gamma)^2}}. \qquad (3)$$

Here, $\delta\beta_L$ is the linear, intrinsic phase birefringence without twist, $\delta\beta'_L$ is the corresponding group birefringence (or DGD for zero twist), and $\delta\beta_C$ and $\delta\beta'_C$ are the twist-induced circular phase and group birefringences, respectively. $\delta\beta_C = g\gamma$ is given by the product of the applied twist rate γ (in turns/m) and the elastooptic rotation coefficient g. In silica fibers, g amounts to 0.14–0.16, depending on the Ge dopant concentration of the fiber.

From a measurement of the DGD as a function of twist, and consecutive fitting of the results with (3), one readily obtains the different unknowns of (3), namely the intrinsic phase and group birefringence in the absence of twist. Note that no a-priori assumptions are made on the different parameters (especially not on the relation between $\delta\beta_L$ and $\delta\beta'_L$): the values for the

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[1]Equation (3) in fact differs from the corresponding equation of [4] due to an erroneous sign.

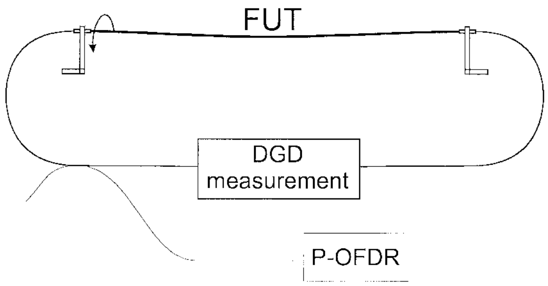

Fig. 1. Diagram of the setup. A connector key has been removed to allow the rotation of the fiber under test.

four fit parameters $\delta\beta_L$, $\delta\beta'_L$, g, and $g\prime = \delta\beta'_c/\gamma$ are allowed to vary freely. However, in order for the fitting algorithm to work properly, reasonable initial values (especially for g) have to be chosen. From an experimental point of view, only standard DGD measurement equipment is required, and the twisting of the fiber under test can be easily achieved by rotating one of the two connectors of that fiber (one can simply remove the corresponding connector key). Fig. 1 shows a sketch of the employed setup.

A first demonstration for a SMF was presented in [4]. In the following, we apply this method to a much larger variety of fibers with beatlengths changing from millimeter to meter range and investigate its precision and applicability for different types of fibers.

III. Evaluation of Measurement Method

The fibers used to test our measurement method, chosen for their well-defined homogeneous birefringences, are summarized in Table I. They are numbered for easy identification, and their geometries (major and minor radius a and b) and refractive index differences (dn) are indicated. Fibers 1 and 2 have an elliptical core, fiber 3 is a Panda fiber, fiber 4 has an elliptical cladding, fibers 5 and 6 are erbium-doped fiber samples with quite large asymmetries, and fiber 7 is a photonic crystal fiber (PCF) with hexagonal hole symmetry (hole diameter: 0.9 μm, core diameter: 3.5 μm).

Fig. 2 shows a measurement result for fiber 1. The fiber length employed was 4 m. For a sufficiently good resolution of the central DGD peak, we rotated the fiber by one full turn between measurements, whereas in the wings, as many as 20 turns can be employed to speed up the measurement. The DGD is measured using the standard Jones matrix eigenanalysis (JME) method [5], typically employing 5-nm steps and a wavelength interval of $\sim$50 nm around 1.55 μm. The points around the minimum DGD values in Fig. 2 (twist of $\sim$10 turns/m) are below the minimum delay of $\sim$5 fs that we can measure with our JME apparatus, and these points have consequently been removed so that they do not falsify the fit. Also, from (3), it can be seen that for large twist values the linear phase birefringence $(\delta\beta_L)$ has only a small influence on the DGD. Consequently, only the central peak is fitted with (3), giving the solid line in Fig. 2. As can be seen, the fit matches quite excellently with the measured points. From the fitting parameters, an intrinsic (i.e., twistless) beatlength of 559 mm is obtained for this fiber.

The reproducibility of the measured data points is found to be quite good. Changing the twist back to a value slightly above zero—where the DGD values are large and change strongly as a function of twist—typically results in deviations of the measured overall delay of about a femtosecond or less, and consequently one finds the same fit values. Moreover, we made a completely independent measurement of fiber 1 using a commercial DGD measurement device from EXFO (FPMD-5600). The precision of this instrument for DGD measurements is better than 0.5% (traced to a NIST calibration artefact). Our DGD measurements of the same fiber where found to agree to within 0.5%, so that we can assume a precision of 1% for our device. However, due to the residual birefringence of the two low birefringence lead fibers (total length of $\sim$ 6 m), we found that the DGD values are systematically increased by $\sim$2 fs. This leads to an error on the beatlength extracted from the fit with (3) that depends on the total amount of DGD and the ratio of group birefringence to phase delay. For the worst parameters still comprised in the measurement range of our setup (see below), a relative error for L_b of 9% has been found with simulations. For the samples measured here, the maximum error is 4% (fiber #1) and <2% for the other ones.

TABLE I
Results of Beatlength Measurements for Seven Investigated Fibers, Using "Twist" Method Presented Here, and two Alternative Methods ("P-OFDR" [6], [7], "Pressure" [8]). δ Gives Deviation of Our Method From Mean Beatlength Found With Alternative Methods. a, b: Major and Minor Axis (Radius) of Core or Cladding (*). Δ: Refractive Index Difference

#	remarks	a (um)	b (um)	Δ	measured Lb (mm) twist	P-OFDR	pressure	δ (%)
1	e-core	1.65	1.43	0.05	**559**	596	570	4.1
2	e-core	7.51	2.4	0.008	**54**	57	55	3.4
3	panda	4.1	4.06	0.005		3.96		
4	e-clading	140*	70*	-	**8.4**	8.1		3.5
5	e-core, Er doped	2	1.82	0.02	**532**	537		0.9
6	e-core, Er doped	2.16	1.69	0.02	**162**	166		2.4
7	PCF	1.8	-	-	**30**	30.7		1.2

For further comparison, two somewhat more standard measurement methods for beatlength extraction have been used. The first one is a coherent, polarization-sensitive reflection measurement (P-OFDR), described in detail in [6] and [7]. Again, twisting of the fiber has to be avoided. As can be seen from Table I, the results of the P-OFDR and the "twist" method typically agree to within 3.5% for the fibers analyzed. The second method works in transmission and consists of analyzing the power through crossed polarizers as a function of the distance at which coupling between the two polarization modes is induced by pressing on the fiber [8]. We again obtained a beatlength agreeing reasonably with the previously obtained values (see Table I).

Looking at the above findings, we can consequently say that the twist method for beatlength extraction works in a precise and reproducible way. However, there are some drawbacks as well. Besides being relatively time-consuming, not all fibers can be measured using this method.

The limits are given by the employed DGD measurement technique (i.e., by the minimum delay it can measure) and by the mechanical strength of the fiber (i.e., by the maximum twist rate that can be applied to the fiber without damaging it).

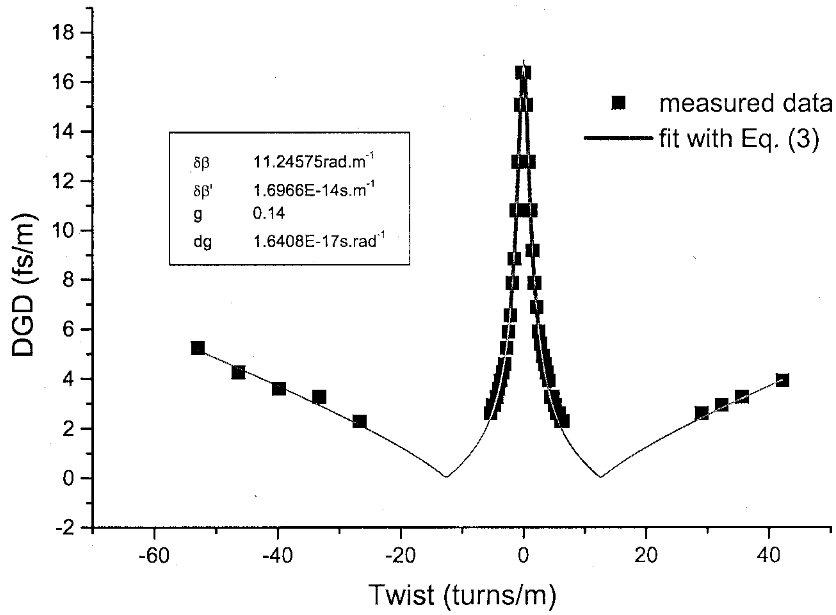

Fig. 2. Experimental results for fiber 1. Square points: measured DGD as a function of the twist rate. Line: fit with (1). The fit parameters $\delta\beta$ and $\delta\beta'$ are the phase and group birefringence of the fiber in the absence of twist.

Typical DGD measurement equipment goes down to about 5 fs. Consequently, we are restricted to fibers with delays in excess of ∼20 fs to get reasonable fits. For practical reasons, the fiber length is limited to about 4 m, so that the maximum measurable beatlength becomes ∼ 1 m. Note that longer fibers (i.e., longer beatlengths) can be measured [9], but it becomes tedious to avoid excess birefringence induced by other means than fiber twist.

To estimate the minimum value of beatlength we can measure, we have to consider that the fibers were found to typically break (at the connector edge) for twist rates in excess of ∼70–80 tr/m. The fiber under test therefore needs to have a birefringence that is sufficiently low to be able to observe enough variation of the DGD within the permissible twist range. We managed to measure a beatlength of 8 mm with our setup, but not 3 mm (fiber 3). It is worth noting that even the PCF (fiber 7), which one would guess to be rather fragile with its fine hole structure, supports large twist rates of >50 tr/m.

Consequently, the twist method for beatlength extraction can be easily applied for fibers with beatlengths of 5 mm to 1 m, with excellent reproducibility and precision.

IV. Ratio of Group Birefringence to Phase Delay

Now that we have seen that for our fiber samples, our measurement method is capable of giving the nontwisted group and phase birefringence with a precision of $\leq$ 1% and $\leq$ 3%, respectively, we can use the obtained results to analyze their ratio, which should be precise to ∼4%.

As seen in Section I, the ratio between group birefringence and phase delay is a measure of how well (1) holds for the corresponding fiber. This can be made more intuitive by introducing the parameters L_b^* and, defined as [10]

$$\mathrm{DGD}^* = \frac{\lambda}{c \cdot L_b} \tag{4}$$

$$L_b^* = \frac{\lambda}{c \cdot \mathrm{DGD}} \tag{5}$$

where DGD* is the group birefringence predicted from the true phase birefringence (i.e., L_b) and (1), whereas L_b^* is the beatlength predicted from the true group birefringence and (1). Consequently, a difference between L_b^* and L_b (or, equivalently, between DGD* and DGD) means that (1) does not hold, and the ratio L_b/L_b^* (which is nothing else than the ratio between group birefringence and phase delay) can be employed to quantify how well (1) holds for a particular fiber.

In the upper part of Table II, the values of L_b, L_b^* and their ratio are given for the previously measured fibers ([1]–[6]). The value of L/L_b^* is found to vary quite a bit, from 1.1 to as much as 2.6, clearly demonstrating that application of the commonly used assumption of (1) would lead to quite erroneous results for most of these fibers. For a more complete picture of when this happens, we thoroughly searched the literature for fibers of which both phase *and* group birefringence were investigated. These examples are given in the lower part of Table II (8–16). The examples found are either PM fibers with beatlengths in the millimeter range or standard fibers from the investigation of Siddiqui. Again, some of the fibers have L_b/L_b^* ratios that strongly deviate from one.

Further analyzing Table II, one finds that for PM fibers with purposefully introduced composite stress elements (fibers 3, 8–11), where consequently the overall birefringence is mainly stress-induced, the ratio is very close to one. This agrees with our own study on this type of fiber [10], where a typical ratio of 1.1 was found both for wavelengths of 1.3 and 1.55 μm. A ∼10% deviation between phase delay and group birefringence therefore seems to be quite generally found in these fibers, independent of wavelength. On the other hand, for fibers with large form birefringence due to large ellipticities of the core or the cladding (fibers 1, 2, 4, 5, 6, 12, 13), the ratio L_b/L_b^* is typically quite different from 1. Moreover, it can strongly

TABLE II
RATIO L_b/L_b^* FOR OUR FIBERS (1-7) AND AS FOUND IN THE LITERATURE (8-16). SEE SECTION IV FOR DETAILS

ref	#	remarks	lambda [nm]	measured Lb [mm]	measured Lb*	ratio
our exp	1	e-core	1550	559	315	1.8
our exp	2	e-core	1550	54	27	2
our exp	3	panda	1550	3.96	3.56	1.1
our exp	4	e-clading	1550	8.4	6.15	1.4
our exp	5	e-core, Er-doped	1550	532	207.5	2.6
our exp	6	e-core, Er-doped	1550	162	62.7	2.5
our exp	7	PCF	1550	30.3	24	1.3
[1]	8	bow-tie	850	1.67	1.45	1.15
[1]	9	bow-tie, e-core	850	2.94	2.52	1.17
[2]	10	stress-induced	838	3.4	3.1	1.1
[2]	11	stress-induced	838	5.5	5.9	0.93
[2]	12	e-core	838	1.1	0.78	1.41
[1]	13	e-core, e-clading	630	0.94	1.42	0.662
[1]	13	e-core, e-clading	850	1.22	1.11	1.1
[1]	13	e-core, e-clading	1300	3.04	0.89	3.42
[4]	14	DSF		3000	2900	1.03
[4]	15	DSF		8400	9400	0.89
[9]	16	SMF	1530	27000	22000	1.23

vary with wavelength, as demonstrated by fiber 13. Finally, for low birefringence fibers (having both low form and stress birefringence; fibers 14–16), the ratio is again close to one, justifying application of (1).

The above findings can be intuitively understood—at least in part—by assuming that the form and stress contributions to the total birefringence can be decoupled, $\overrightarrow{\delta\beta} = \overrightarrow{\delta\beta}_{\rm geo} + \overrightarrow{\delta\beta}_{\rm stress}$ [11], [12]. Although this is certainly not completely true [e.g., [13]], it is still a good approximation of reality. The two individual contributions to the phase birefringence are sketched out in Fig. 3 as a function of the normalized frequency V. The phase birefringence induced by stress is linear with V, whereas the form contribution is a more complicated function of V.

The group birefringence is the derivative of the phase birefringence with frequency, and therefore proportional to $d(\delta\beta)/dV$. Consequently, the stress contribution to the group birefringence is independent of V and equal to the phase delay $\delta\beta/\omega$, leading to a ratio of one. If form birefringence is important as well, as for the second group of fibers (1, 2, 4, 5, 6, 12, 13), things become more subtle as the overall phase birefringence stems both from purely geometrical effects (waveguide ellipticity) and from the stress induced by this asymmetry and the difference of dilatation between core and cladding. Their respective weight depends on the characteristics of the waveguide and on the normalized frequency V. Due to the shape (see Fig. 3) and relative weight of the form-induced phase birefringence, the corresponding group birefringence, and with that the ratio L_b/L_b^*, will typically deviate from one for these fibers, although at specific V it can still amount to ~1. This is nicely illustrated by the example of fiber 13 in Table II, where, as a function of wavelength (and therefore of V), values of 0.66, 1.1, and 3.4 are found.

The above considerations are, however, not explaining why for low birefringence fibers (14–16 in Table II), $L_b/L_b^* \sim 1$. From [14], where the stress distribution of an SMF-28 fiber was measured, a stress-induced beatlength of ~50 m is obtained. Comparing this value to the overall (i.e., stress *and* form induced) beatlength of a typical standard fiber ($L_b \sim 20 - 30$ m),

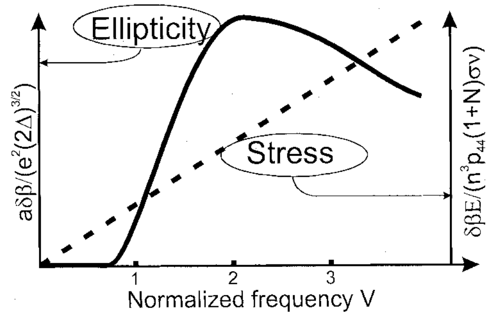

Fig. 3. Normalized phase birefringence induced by stress (dashed line) or fiber geometry (continuous line) as a function of normalized frequency ($V = 2\pi a n\sqrt{2\Delta}\nu$). a: core radius, n: refractive index, Δ : refractive index difference, ν: optical frequency, e: core ellipticity, p_{44}: component of the strain-elastooptic tensor of the core, N: Poisson's ratio, and E: Young's modulus.

one finds that there are both form and stress contributions in such fibers, with about the same relative weight. Consequently, one would expect the ratio L_b/L_b^* to deviate from one.

Apparently, the above model is too simple to correctly explain things for standard fibers. Also, it seems extremely difficult to correctly predict phase and group birefringence from measurements of the fiber parameters alone (index profile, stress distribution; an overview of different models can be found in [15]). No conclusive examples are found in the literature, just observations that more complex (i.e., complete) models would have to be used to correctly account for the interplay between form and stress contributions [16].

V. CONCLUSION

A quite novel measurement method to extract the phase and group birefringence of an optical fiber has been presented. Its main advantage, besides requiring only standard DGD measurement equipment, is that it assures that one obtains the twistless intrinsic values. Both phase and group birefringence can change dramatically in the presence of the small twists that are typically present when no special care is taken. The reproducibility and accuracy are found to be excellent for a large variety of different fiber types. For an easy implementation of the method, however, the beatlength has to be in the range of 5 mm–1 m.

Using this measurement tool, the intrinsic phase and group birefringences have been measured for various fibers. Using our results and some previously published data, one finds that the commonly used assumption that phase delay and group birefringence in an optical fiber are equal is not justified in all fiber types. Especially in fibers with nonnegligible contributions of form birefringence, and depending on the normalized frequency, one finds factors as large as three between group birefringence and phase delay. For PM fibers based on stress elements and standard, low-birefringence fibers at 1.55 μm, phase delay and group birefringence coincide to within 10%.

These findings can be mostly explained using a simple and intuitive model where the contributions of form- and stress-induced birefringence are considered to be independent. It does not explain, however, why phase delay and group birefringence seem to be equal in standard fibers.

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